

System-environment dynamics of X-type states in noninertial frames

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Abstract

The system-environment dynamics of noninertial systems is investigated. It is shown that for the amplitude damping channel: (i) the biggest difference between the decoherence effect and the Unruh radiation on the dynamics of the entanglement is the former only leads to entanglement transfer in the whole system, but the latter damages all types of entanglement; (ii) the system-environment entanglement increases and then declines, while the environment-environment entanglement always increases as the decay parameter p increases; and (iii) the thermal fields generated by the Unruh effect can promote the sudden death of entanglement between the subsystems while postpone the sudden birth of entanglement between the environments. It is also found that there is no system-environment and environment-environment entanglements when the system coupled with the phase damping environment.

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I. INTRODUCTION

It is well known that quantum entanglement is a key resource for the implementation of many quantum information protocols [1], such as quantum communication, quantum cryptography, quantum teleportation and computation. However, although many efforts have been made to the study of the properties of entanglement, the good understanding of such a resource is only limited in inertial systems. Doubtlessly, the research of the entanglement behaviors in a relativistic setting will not only provide a more complete framework for the quantum information theory, but also play an important role in the understanding of the entropy and information paradox [2, 3] of black holes. In addition, it is also closely related to the implementation of quantum computation with observers in arbitrary relative motion [4] and the study of the physical bounds of quantum information processing tasks. As a result, there are an increasing number of articles discussing the entanglement in the relativistic setting, in particular on how the Unruh and Hawking effect influence the degree of entanglement [4–19].

On the other hand, real quantum systems are necessarily subjected to their environments, and these reciprocal interactions often result in loss of quantum coherence and entanglement. Such a process is usually called quantum decoherence [20, 21], which has been widely investigated. It is often stated that the decoherence causes the system to become entangled with its environment, and then the dynamics of the system is non-unitary. It plays a fundamental role in the description of the quantum-to-classical transition [22, 23] and has been successfully applied in the cavity QED [24] and the ion trap experiments [25].

In this paper we will discuss the system-environment dynamics for X-type state of the Dirac fields in a noninertial frame. As far as we known, either the entanglement behaviors of the X-type states or the system-environment dynamics has not been investigated in non-inertial frames yet. The Dirac field, as shown in Refs. [26–28], from an inertial perspective, can be described by a superposition of Unruh monochromatic modes $|0\rangle_U = \bigotimes_{\omega} |0_{\omega}\rangle_U$ and $|1\rangle_U = \bigotimes_{\omega} |1_{\omega}\rangle_U$, with

$$|0_{\omega}\rangle_U = \cos r |0_{\omega}\rangle_I |0_{\omega}\rangle_{II} + \sin r |1_{\omega}\rangle_I |1_{\omega}\rangle_{II}, \quad (1)$$

and

$$|1_{\omega}\rangle_U = |1_{\omega}\rangle_I |0_{\omega}\rangle_{II}, \quad (2)$$

where $\cos r = (e^{-2\pi\omega c/a} + 1)^{-1/2}$, a is the acceleration of the observer, ω is frequency of the Dirac particle, and c is the speed of light in vacuum. We assume that two observers, Alice and Rob, share an entangled X-type initial state. Rob detects a single Unruh mode and Alice detects a monochromatic Minkowski mode of the Dirac field. Considering that an accelerated observer must remain in either region I or II due to these two regions are causally disconnected, i. e., an observer in region I can't access to the field modes in the causally disconnected region II , we should trace over the inaccessible modes.

The outline of the paper is as follows. In Sec. II we recall some concepts from the view of the quantum information theory, in particular the theory of open quantum systems. In Sec. III we investigate the system-environment dynamics of X-type states in the noninertial frames. We summarize and discuss our conclusions in the last section.

II. OPEN SYSTEM DYNAMICS

Let us start by briefly review the theory of open quantum systems (for details see Ref. [24]). The total evolution of a system plus the environment can be described by $U_{SE}(\rho_S \otimes |0\rangle_E \langle 0|)U_{SE}^\dagger$, where U_{SE} is the evolution operator for the total state, and $|0\rangle_E$ represents the initial state of the environment. By tracing over the degrees of freedom of the environment, we can get the evolution of the system

$$\begin{aligned} L(\rho_S) &= \text{Tr}_E[U_{SE}(\rho_S \otimes |0\rangle_E \langle 0|)U_{SE}^\dagger] \\ &= \sum_{\mu} {}_E \langle \mu | U_{SE} | 0 \rangle_E \rho_S {}_E \langle 0 | U_{SE}^\dagger | \mu \rangle_E, \end{aligned} \quad (3)$$

where $|\mu\rangle_E$ is the orthogonal basis for the environment, and the operator L describes the evolution of the system. Eq. (3) can also be expressed as

$$L(\rho_S) = \sum_{\mu} M_{\mu} \rho_S M_{\mu}^\dagger, \quad (4)$$

where

$$M_{\mu} \equiv {}_E \langle \mu | U_{SE} | 0 \rangle_E, \quad (5)$$

are the Kraus operators [29, 30]. There are at most d^2 independent Kraus operators, where d is the dimension of the system [31, 32]. Together with Eq. (5), the dynamical evolution of the complete system-environment state can be also given by the following map [31] :

$$\mathbf{U}_{SE} |\xi_l\rangle_S \otimes |0\rangle_E = \sum_k M_k |\xi_l\rangle_S \otimes |k\rangle_E.$$

with

$$\begin{aligned}
|\xi_1\rangle_S \otimes |0\rangle_E &\rightarrow M_0|\xi_1\rangle_S \otimes |0\rangle_E + \cdots + M_{d^2-1}|\xi_1\rangle_S \otimes |d^2-1\rangle_E \\
|\xi_2\rangle_S \otimes |0\rangle_E &\rightarrow M_0|\xi_2\rangle_S \otimes |0\rangle_E + \cdots + M_{d^2-1}|\xi_2\rangle_S \otimes |d^2-1\rangle_E \\
&\vdots \\
|\xi_d\rangle_S \otimes |0\rangle_E &\rightarrow M_0|\xi_d\rangle_S \otimes |0\rangle_E + \cdots + M_{d^2-1}|\xi_d\rangle_S \otimes |d^2-1\rangle_E,
\end{aligned} \tag{6}$$

where $\{|\xi_l\rangle_S\}$ ($l = 1, \dots, d$) is the complete basis for the system.

III. SYSTEM-ENVIRONMENT DYNAMICS OF ENTANGLEMENT

We assume that Alice and Rob share a X-type initial state

$$\rho_{AR} = \frac{1}{4} \left(I_{AR} + \sum_{i=0}^3 c_i \sigma_i^{(A)} \otimes \sigma_i^{(R)} \right), \tag{7}$$

where I_{AR} is the identity operator in the Hilbert space of the two qubits A and R , $\sigma_i^{(A)}$ and $\sigma_i^{(R)}$ are the Pauli operators of the qubits A and R , and c_i ($0 \leq |c_i| \leq 1$) are real numbers satisfying the unit trace and positivity conditions of the density operator ρ_{AR} . Eq. (7) represents a class of states including the general initial state, the Werner initial state ($|c_1| = |c_2| = |c_3| = c$), and the Bell basis ($|c_1| = |c_2| = |c_3| = 1$). After the coincidence of Alice and Rob, Alice stays stationary while Rob moves with uniform acceleration a . Using Eqs. (1) and (2), we can rewrite Eq. (7) in terms of Minkowski modes for Alice and Rindler modes for Rob. Since Rob is causally disconnected from the region II , the only information which is physically accessible to the observers is encoded in the Minkowski modes A described by Alice and the Rindler modes \tilde{R} described by Rob. Taking the trace over the modes in region II , we obtain

$$\rho_{A\tilde{R}} = \frac{1}{4} \begin{bmatrix} (1+c_3)\cos^2 r & 0 & 0 & c^- \cos r \\ 0 & (1+c_3)\sin^2 r + (1-c_3) & c^+ \cos r & 0 \\ 0 & c^+ \cos r & (1-c_3)\cos^2 r & 0 \\ c^- \cos r & 0 & 0 & (1-c_3) + (1+c_3)\sin^2 r \end{bmatrix},$$

where $|mn\rangle = |m\rangle_A |n\rangle_{\tilde{R}}$, $c^+ = c_1 + c_2$, and $c^- = c_1 - c_2$.

A. Amplitude damping

Now we consider both Alice and Rob's qubits under the amplitude damping environment, and the environment acts independently on Alice and Rob's states. From Eq. (6) we find that the action of the amplitude damping channel over one qubit can be represented by the following phenomenological map [33]

$$|0\rangle_i|0\rangle_{E_i} \rightarrow |0\rangle_i|0\rangle_{E_i} \quad (8a)$$

$$|1\rangle_i|0\rangle_{E_i} \rightarrow \sqrt{q_i}|1\rangle_i|0\rangle_{E_i} + \sqrt{p_i}|0\rangle_i|1\rangle_{E_i}, \quad (8b)$$

where $q_i = 1 - p_i$, and $|0\rangle_i$ ($i = A, \tilde{R}$) are the ground and $|1\rangle_i$ are the excited qubit states of the $A\tilde{R}$ system. $|0\rangle_{E_i}$ and $|1\rangle_{E_i}$ describe the states of the environment with no excitation and one excitation of its modes, respectively. We use p_i ($0 \leq p_i \leq 1$) to describe these probabilities as a parametrization of time. Here we only consider the simplest case in which all the subsystems are embedded in the same environments, i.e., $p_A = p_{\tilde{R}} = p$ [31].

The total initial density operator of the whole system can be described as

$$\rho_{A\tilde{R}E_AE_{\tilde{R}}} = \rho_{A\tilde{R}} \otimes \rho_{E_AE_{\tilde{R}}}, \quad (9)$$

where ρ_{E_i} is the vacuum state of the environments. Now by use of Eqs. (8a), (8b) and (9), we can compute the entanglement of the total density matrix $\rho_{A\tilde{R}E_AE_{\tilde{R}}}$ and discuss how it evolves. But here we are interesting in the entanglement dynamics of the bipartite subsystems (especially the system-environment dynamics), we only need to consider the corresponding bipartite reduced matrixes. The reduced-density matrix of the inertial subsystem A and the noninertial subsystem \tilde{R} , obtained by taking the partial trace of $\rho_{A\tilde{R}E_AE_{\tilde{R}}}$ over the degrees of freedom of the environment $\rho_{A\tilde{R}}(a) = \text{Tr}_{E_AE_{\tilde{R}}}[\rho_{A\tilde{R}E_AE_{\tilde{R}}}]$, is given by

$$\rho_{A\tilde{R}}(a) = \frac{1}{4} \begin{bmatrix} \alpha & 0 & 0 & qc^- \cos r \\ 0 & q(\gamma + \beta p) & qc^+ \cos r & 0 \\ 0 & qc^+ \cos r & q[\varepsilon + \beta p] & 0 \\ qc^- \cos r & 0 & 0 & \beta q^2 \end{bmatrix}, \quad (10)$$

where $\alpha = \epsilon + p(2\varepsilon + \beta p)$, $\beta = (1 + c_3) + \sin^2 r(1 - c_3)$, $\gamma = (1 - c_3) + \sin^2 r(1 + c_3)$, $\varepsilon = (1 - c_3) \cos^2 r$ and $\epsilon = (1 + c_3) \cos^2 r$.

We are especially interested in the dynamical evolution of entanglement between the non-inertial subsystem \tilde{R} and its environment $E_{\tilde{R}}$. The corresponding reduced-density operator can be obtained by tracing over the degrees of freedom of subsystem A and environment E_A

$$\rho_{\tilde{R}E_{\tilde{R}}}(a) = \frac{1}{4} \begin{bmatrix} 2 \cos^2 r & 0 & 0 & 0 \\ 0 & p(\beta + \gamma) & \sqrt{pq}(\beta + \gamma) & 0 \\ 0 & \sqrt{pq}(\beta + \gamma) & q(\beta + \gamma) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (11)$$

and similarly, the state of the noninertial subsystem \tilde{R} and the environment E_A reads

$$\rho_{\tilde{R}E_A}(a) = \frac{1}{4} \begin{bmatrix} \delta & 0 & 0 & \sqrt{pq}c^- \cos r \\ 0 & q(\gamma + \beta q) & \sqrt{pq}c^+ \cos r & 0 \\ 0 & \sqrt{pq}c^+ \cos r & p[\varepsilon + \beta p] & 0 \\ \sqrt{pq}c^- \cos r & 0 & 0 & \beta pq \end{bmatrix}, \quad (12)$$

where $\delta = \epsilon + q[\beta p + \varepsilon] + \gamma p$.

Again, by tracing out the system degrees of freedom, we get the bipartite matrix of the partition $E_A E_{\tilde{R}}$

$$\rho_{E_A E_{\tilde{R}}} = \frac{1}{4} \begin{bmatrix} \chi & 0 & 0 & pc^- \cos r \\ 0 & p(\gamma + \beta q) & pc^+ \cos r & 0 \\ 0 & pc^+ \cos r & p[\varepsilon + \beta q] & 0 \\ pc^- \cos r & 0 & 0 & \beta p^2 \end{bmatrix}, \quad (13)$$

where $\chi = \epsilon + q[\beta q + \varepsilon + \gamma]$.

The entanglement of a two-qubits mixed state ρ in a noisy environments can be quantified conveniently by the concurrence, which is defined as [34, 35]

$$C(\rho) = \max \{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad \lambda_i \geq \lambda_{i+1} \geq 0, \quad (14)$$

where λ_i are the square roots of the eigenvalues of the matrix $\rho \tilde{\rho}$, $\tilde{\rho} = (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y)$ is the “spin-flip” matrix. But fortunately, due to the density matrixes from (10) to (13) have X -type structures, here we have a simpler expression for the concurrence [33]

$$C(\rho) = 2 \max \left\{ 0, \tilde{C}_1(\rho), \tilde{C}_2(\rho) \right\}, \quad (15)$$

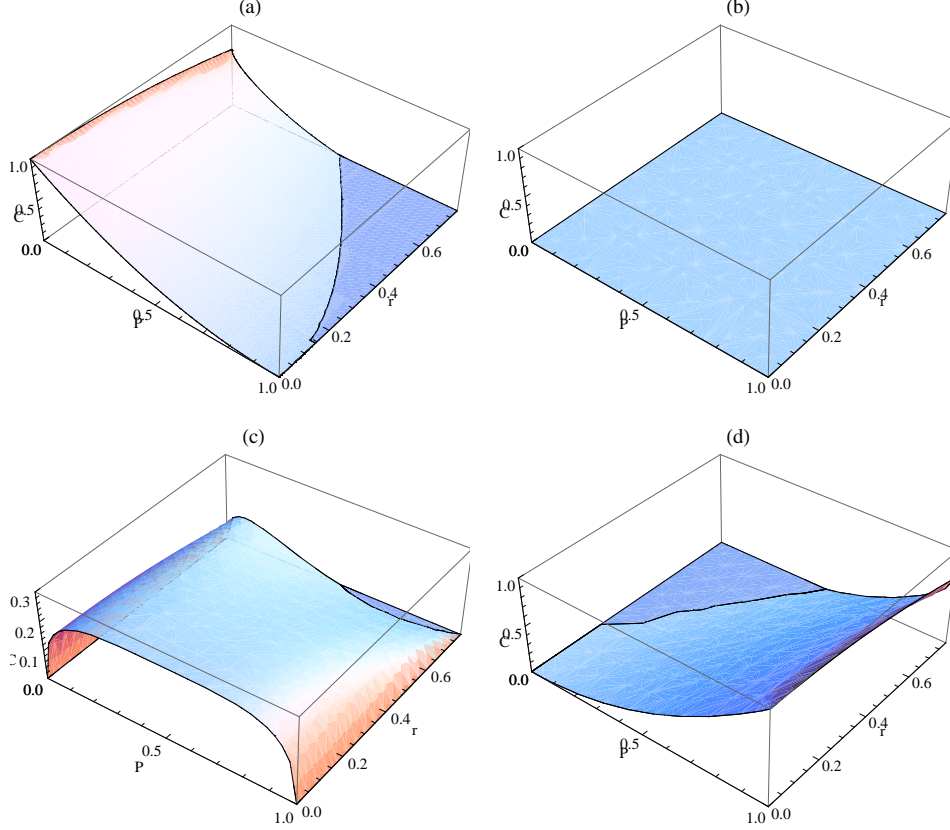


FIG. 1: (Color online) Entanglement dynamics for the amplitude-damping channel, considering the cases of Bell initial states ($|c_1| = |c_2| = |c_3| = 1$), for bipartite states: (a) $A\tilde{R}$, (b) $\tilde{R}E_{\tilde{R}}$, (c) $\tilde{R}E_A$, and (d) $E_A E_{\tilde{R}}$ respectively.

with $\tilde{C}_1(\rho) = \sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{22}\rho_{33}}$ and $\tilde{C}_2(\rho) = \sqrt{\rho_{23}\rho_{32}} - \sqrt{\rho_{11}\rho_{44}}$, where ρ_{ij} are elements of the density matrix ρ . Then we can easily obtain analytical expressions of the concurrence for the bipartite matrixes from (10) to (13) and plot them in the Figs. 1 and 2. Note that the entanglement of bipartite subsystems $AE_A(a)$ and $AE_{\tilde{R}}(a)$ are not presented in these figures. This is due to the fact that the environments E_A and $E_{\tilde{R}}$ are symmetrical, thus the density matrix representing the bipartite subsystem $AE_A(a)$ is similar to that of the bipartite subsystem $\tilde{R}E_{\tilde{R}}(a)$, and leading to a similar dynamic. In fact, we can prove that the concurrence $C_{AE_A(a)} = C_{\tilde{R}E_{\tilde{R}}(a)} = 0$. The same similarity exists between the bipartite subsystems $AE_{\tilde{R}}(a)$ and $\tilde{R}E_A(a)$.

Fig. 1 shows the dynamics of the entanglement for all the partitions $A\tilde{R}$, $\tilde{R}E_{\tilde{R}}$, $\tilde{R}E_A$, and $E_A E_{\tilde{R}}$, when $|c_1| = |c_2| = |c_3| = 1$. It is shown that the entanglement of the subsystem $A\tilde{R}$ suffers sudden death (SD) at some certain acceleration parameter r and decay

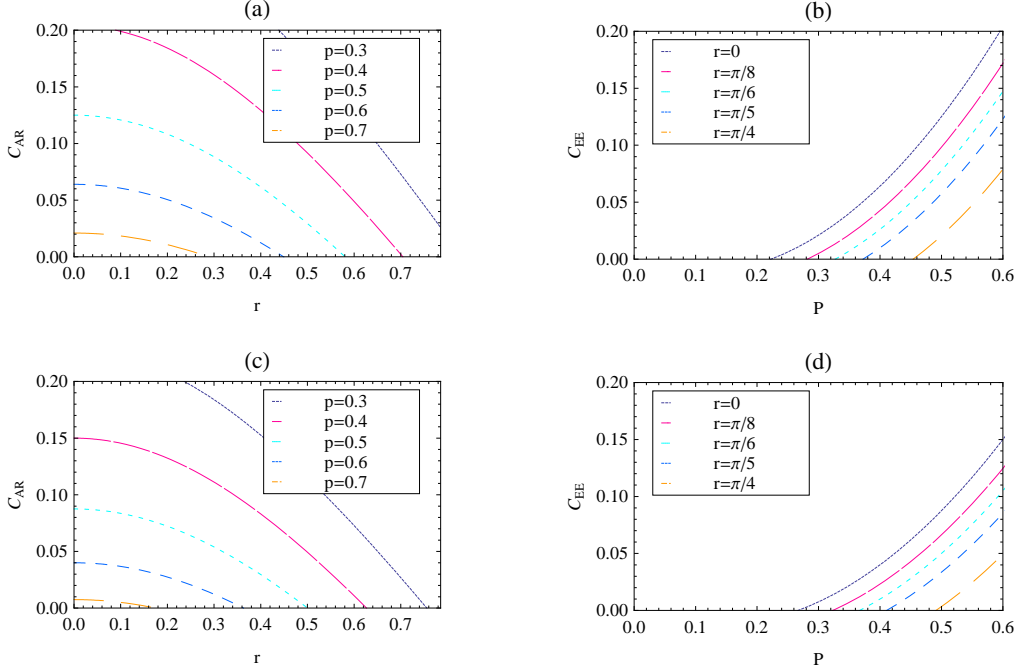


FIG. 2: (Color online) Entanglement dynamics for the amplitude-damping channel, considering the cases: (a) bipartition $A\tilde{R}$ ($|c_1| = |c_2| = |c_3| = 0.8$), (b) bipartition $E_A E_{\tilde{R}}$ ($|c_1| = |c_2| = |c_3| = 0.8$), (c) bipartition $A\tilde{R}$ ($|c_1| = 0.7, |c_2| = 0.9, |c_3| = 0.4$), and (d) bipartition $E_A E_{\tilde{R}}$ ($|c_1| = 0.7, |c_2| = 0.9, |c_3| = 0.4$) respectively.

parameter p . We also note that the entanglement between the noninertial subsystem \tilde{R} and its environment $E_{\tilde{R}}$ always equals to zero for any r and p . However, the interaction between the system and environment generates system-environment entanglement between the noninertial subsystem \tilde{R} and the environment E_A . As the decay parameter p increases, the system-environment entanglement of $\tilde{R}E_A$ increases firstly and then decreases quickly. However, as the acceleration increases, such entanglement always decreases and appears SD at some larger accelerations. At the same time, it is interesting to note that this interaction also generates environment-environment entanglement between the environments E_A and $E_{\tilde{R}}$, and such entanglement exhibits entanglement sudden birth (SB) [36] at some certain r and p . It is worthy to notice that when $p = 0$, the entanglement of the system (Fig. 1a) is 1 while the system-environment entanglement (Fig. 1c) and environment-environment entanglement (Fig. 1d) are zero. However, when $p = 1$, there is only environment-environment entanglement. That is to say, at first the entanglement of the system was transferred to system-environment and environment-environment entanglement, but finally all these lost

entanglement were transferred to the environment degrees of freedom. Thus, we can see that the system-environment entanglement increases, reaches a maximum, and then declines. Then we conclude that the most different between the decoherence and Unruh effect on the dynamics of the entanglement in noninertial frames is that the former only leads to entanglement transfer in the whole system, while the latter damages not only entanglement of the system, but also system-environment and environment-environment entanglement. It is also shown that the SD of entanglement in Figs. 1a and 1c occurs almost as the same time as the SB of entanglement in Fig. 1d for very large r . In fact, it is very easy to show that SB might be occurring before, simultaneously with or even after SD, depending on different initial states. For example, one can plot a similar figure as Fig. 1 for the case of a Werner initial state with $|c_1| = |c_2| = |c_3| = 0.7$ and find that SB occurs much later than SD.

Fig. 2 shows how the acceleration and decay parameter affect the SD and SB of the entanglement for Werner ($|c_1| = |c_2| = |c_3| = 0.8$) and general ($|c_1| = 0.7, |c_2| = 0.9, |c_3| = 0.4$) initial states. We note that for both of these two cases: (i) the monotonous decrease of entanglement of the system $A\tilde{R}$ as the acceleration increase can attribute to the thermal fields generated by the Unruh effect; (ii) a larger p leads to an earlier SD as the acceleration increases; and (iii) the entanglement between the environments E_A and $E_{\tilde{R}}$ always increases as time p increases but decreases as the acceleration increases. However, it is worthy to note that a larger acceleration results in an earlier SD of entanglement between the subsystems A and \tilde{R} but a later SB of entanglement between the environments E_A and $E_{\tilde{R}}$. The thermal fields generated by the Unruh effect can promote SD but postpone SB of entanglement in noninertial frames. We also note that in the case of $p = 0.3$ (and naturally when $p < 0.3$), for a Werner initial state the entanglement didn't tends to zero even the acceleration approaches to infinity, which is quite different from the general initial state case. It seems that the form of initial state also plays an important role in the system-environment dynamics of the entanglement in noninertial frames.

B. Phase damping channel

In this subsection we discuss the dynamics of system-environment entanglement under the phase-damping channel, which describes the loss of quantum coherence without losing

energy [37]. The map of this channel on a one-qubit system is given by

$$|0\rangle_i|0\rangle_{E_i} \rightarrow |0\rangle_i|0\rangle_{E_i} \quad (16a)$$

$$|1\rangle_i|0\rangle_{E_i} \rightarrow \sqrt{q_i}|1\rangle_i|0\rangle_{E_i} + \sqrt{p_i}|1\rangle_i|1\rangle_{E_i}, \quad (16b)$$

where $|0\rangle_i$ ($i = A, \tilde{R}$) .

The reduced-density operator for the partition $A\tilde{R}$, obtained by taking the partial trace of $\rho_{A\tilde{R}E_AE_{\tilde{R}}}(p)$ over the degrees of freedom of the environment, is given by

$$\rho_{A\tilde{R}}(p) = \frac{1}{4} \begin{bmatrix} \epsilon & 0 & 0 & qc^- \cos r \\ 0 & \gamma & qc^+ \cos r & 0 \\ 0 & qc^+ \cos r & \epsilon & 0 \\ qc^- \cos r & 0 & 0 & \beta \end{bmatrix}, \quad (17)$$

For the bipartite subsystems $\tilde{R}E_{\tilde{R}}$ and $\tilde{R}E_A$, the reduced-density operators are found to be

$$\rho_{\tilde{R}E_{\tilde{R}}}(p) = \frac{1}{4} \begin{bmatrix} 2\cos^2 r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & q(\beta + \gamma) & \sqrt{pq}(\beta + \gamma) \\ 0 & 0 & \sqrt{pq}(\beta + \gamma) & p(\beta + \gamma) \end{bmatrix}. \quad (18)$$

and

$$\rho_{\tilde{R}E_A}(p) = \frac{1}{4} \begin{bmatrix} \epsilon q + \epsilon & 0 & \sqrt{pq}\epsilon & 0 \\ 0 & \gamma + \beta q & 0 & \beta\sqrt{pq} \\ \sqrt{pq}\epsilon & 0 & \epsilon p & 0 \\ 0 & \beta\sqrt{pq} & 0 & \beta p^2 \end{bmatrix}. \quad (19)$$

Similarly, for the partition $E_AE_{\tilde{R}}(p)$, by tracing out the system degrees of freedom, we obtain

$$\rho_{E_AE_{\tilde{R}}}(p) = \frac{1}{4} \begin{bmatrix} \varpi & \sqrt{pq}(\gamma + \beta q) & \sqrt{pq}(\epsilon + \beta q) & \beta pq \\ \sqrt{pq}(\gamma + \beta q) & (\gamma + \beta q)p & \beta pq & \beta p\sqrt{pq} \\ \sqrt{pq}(\epsilon + \beta q) & \beta pq & p[\epsilon + \beta q] & \beta p\sqrt{pq} \\ \beta pq & \beta p\sqrt{pq} & \beta p\sqrt{pq} & \beta p^2 \end{bmatrix}, \quad (20)$$

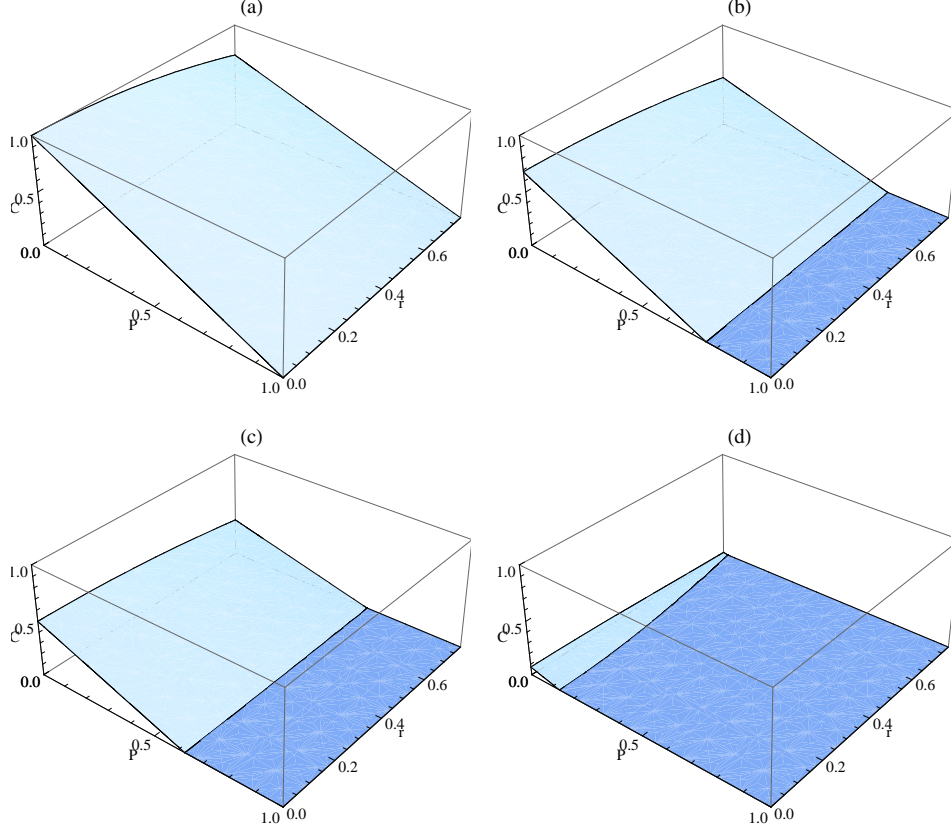


FIG. 3: (Color online) Entanglement dynamics of bipartition $A\tilde{R}$ for the phase damping channel, considering the cases: (a) Bell initial states, (b) Werner initial states $|c_1| = |c_2| = |c_3| = 0.9$, (c) Werner initial states $|c_1| = |c_2| = |c_3| = 0.8$, and (d) general initial states $|c_1| = 0.6, |c_2| = 0.5, |c_3| = 0.3$ respectively.

where $\varpi = \epsilon + q[\beta q + \varepsilon + \gamma]$. We can see that only the density matrix Eq. (17) has an X -type structure. By using of the Peres separability criterion [38], we find that there is no entanglement in bipartite states from Eqs. (18) to (20). In other words, the interaction between system and environment didn't generate bipartite system-environment and environment-environment entanglement in the phase damping case.

Fig. 3 shows how the acceleration and decay parameter change the entanglement of the system $A\tilde{R}$ for different initial states. We find again that the entanglement of the system, as well as the system-environment and environment-environment entanglement decrease as r increases for fixed p , which is as same as the amplitude damping case. We can see that (i) for a Bell state ($|c_1| = |c_2| = |c_3| = 1$), there is no SD of entanglement, which is quite different from the amplitude-damping case; (ii) for Werner states ($|c_1| = |c_2| = |c_3| = 0.9$)

and ($|c_1| = |c_2| = |c_3| = 0.8$) the SD always appears as r and p increase; and (iii) for general initial states ($|c_1| = 0.6, |c_2| = 0.5, |c_3| = 0.3$), the SD of entanglement appears very early. Now we can safely come to the conclusion that the form of initial state do plays an important role in the system-environment dynamics of entanglement in noninertial frames.

IV. SUMMARY

We investigated the system-environment dynamics in a noninertial frame when both the noninertial and inertial subsystems coupled with environments. It is shown that for the amplitude damping channel, only the entanglement between subsystem \tilde{R} and its environment $E_{\tilde{R}}$ equals to zero. However, there is no entanglement in bipartite states $\tilde{R}E_{\tilde{R}}$, $\tilde{R}E_A$, and $E_AE_{\tilde{R}}$ when the system coupled with the phase damping environment. It is found that the biggest difference between the decoherence and Unruh effect on the dynamics of the entanglement in noninertial frames is the former only leads to entanglement transfer in the whole system, while the latter damages not only entanglement of the system, but also system-environment and environment-environment entanglement. In the amplitude damping case, the entanglement of the system $A\tilde{R}$ suffers SD at some certain p and r for any initial states. However, when the system is under the phase damping channel, for a Bell initial state the entanglement tends to zero only when the acceleration approaches to infinity or $p = 1$, which is quite different from the amplitude-damping case. At the same time, it is found that for the amplitude damping, the interaction between system and environment generates bipartite system-environment entanglement between the noninertial subsystem \tilde{R} and the environment E_A . As the decay parameter p increases, the system-environment entanglement increases firstly and then decreases quickly. However, as the acceleration increases, the system-environment entanglement always decreases and appears a SD at some larger accelerations. It is interesting to note that when the system coupled with the amplitude damping environment, such an interaction also generates entanglement between the environments E_A and $E_{\tilde{R}}$, and such entanglement exhibits a SB. It is worthy to notice that a larger acceleration results in an earlier SD of entanglement between the subsystems A and \tilde{R} but a later SB of entanglement between the environments E_A and $E_{\tilde{R}}$. The thermal fields generated by the Unruh effect can promote SD but postpone SB of entanglement in noninertial frames. We also find that the form of initial state plays an important role in the

system-environment dynamics of entanglement in noninertial frames.

Acknowledgments

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